From Quantum Field Theory to Classical Transport Theory*

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The one loop effective action in quantum field theory can be expressed as a quantum mechanical path integral over world lines, with internal symmetries represented by Grassmanian variables. In this work, we developed a real time, many body, world line formalism for the one loop effective action. In particular, we studied hot QCD and obtained the classical transport equations which, as Litim and Manuel have shown, reduce in the appropriate limit to the non–Abelian Boltzmann–Langevin equation first obtained by Bödeker. In the Vlasov limit, the classical kinetic equations are those that correspond to the hard thermal loop effective action.

In classical kinetic theory, a covariant formalism can be obtained in terms of phase space averages over the trajectories of particle world lines. In quantum field theory, there are many instances at finite temperature and density where classical ideas are relevant, and where a classical kinetic picture would be useful. However, it is not immediately apparent how one recovers the classical world line picture directly from quantum field theory. This is especially problematic in theories with internal symmetries.

Fortunately, in the last decade, there has been a considerable body of work relating the one loop effective action in quantum field theory to quantum mechanical path integrals over point particle Lagrangians. The connection between fields and particles is of course relatively ancient. It goes back to the works of Feynman and Schwinger.

Our starting point is the QCD one-loop effective action

$$S_{\text{eff}} = \ln \text{Det} D^2 \tag{1}$$

where $D = \partial - igA$ is the covariant derivative. By using the Schwinger proper time method (generalized to deal with internal degrees of freedom), we were able to re-write it as a classical Lagrangian

$$L = -m\sqrt{\dot{x}_\mu \dot{x}^\mu} + i\lambda_a^\dagger D_{ab}\lambda_b \, . \tag{2}$$

Here the $\lambda_a(\tau)$ with $a = 1, \cdots, N$ are Grassmanian dynamical variables, and $D_{ab}\lambda_b = \dot{\lambda}_a + ig\dot{x}^\mu A_\mu^\alpha T_{ab}^\alpha \lambda_b$. The Euler-Lagrange equations of motion are deduced from this Lagrangian are precisely the equations written down nearly thirty years ago by S.K. Wong.

The corresponding color current is then,

$$J^{\mu,\alpha}_{A,\xi}(x) = g \int d^4p \, dQ \, p^\mu \, Q^\alpha \, f_{A,\xi}(x,p,Q) \, . \tag{3}$$

where A is the soft background Yang-Mills field and ξ is the classical noise corresponding to the distribution of the initial state. The classical phase space density $f_{A,\xi}(x,p,Q)$ satisfies the Vlasov equation used by Kelly et al. and also Litim and Manuel to derive the so-called Hard Thermal Loop action.

In this way, we show how in the real time many body world line formalism the classical transport theory of Kelly et al., as improved by Litim and Manuel, may be obtained from the one loop effective action in QCD. We obtain the same set of transport equations they do. Since our results are derived from the QCD one loop effective action, one can, in principle, go further. Research in that direction as well as the link to the small x physics is being pursued by us.

Footnotes and References

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